Lesson 27 Partial Derivates

I. Geometry in 3D

II. Notation

III. Examples

Test 3 - See Brightspace New Room!

Time Change - Fall Back this weekend

Comments on level curves

$Ax + By = C$ line no squares

$Ax \pm By^2 = C$, $Ax^2 \pm By = C$ } parabolas — x and y^2 or y and x^2

$A(x \pm a) \pm B(y \pm b)^2 = C$, $A(x \pm a)^2 + B(y \pm b) = C$ } parabolas

$x^2 + y^2$

Look @ coeffs in front of $x^2 + y^2$

$A(x \pm a)^2 + B(y \pm b)^2 = C$ } ellipse

positive

$A(x \pm a)^2 + A(y \pm b)^2 = C$ } circle

$A(x \pm a)^2 - B(y \pm b)^2 = C$, $-A(x \pm a)^2 + B(y \pm b)^2 = C$ } hyperbolas

Partial Derivatives

I Geometry in 3D

2D - geometry for derivatives

formulas for slopes of tan lines

Geogebra pictures

Key idea: Steepness is is different at a point on
(slope)
the curve depending on the direction
it travels.

$$z = \frac{x^2}{4} - \frac{y^2}{9}$$

What if an ant sticks to a path where $x = 5$?

$$z = \frac{25}{4} - \frac{y^2}{9}$$

$$\frac{dz}{dy} = -\frac{2y}{9}$$

What if ant sticks to path where $y = 3$?

$$z = \frac{x^2}{4} - 1$$

$$\frac{dz}{dx} = \frac{2}{4}x$$

There are other directions ant could go, but for now we stick with the idea that the ant always decides

$x = c$ some constant

OR $y = c$

In fact, it did not matter that we chose $x=5$ path. If I chose any $x=C$ path

$$z = \frac{c^2}{4} - \frac{y^2}{9}$$

$$\frac{dz}{du} = -\frac{2}{9}y$$

Similarly, for any $y=C$ path

$$z = \frac{x^2}{4} - \frac{c^2}{9}$$

$$\frac{dz}{dx} = \frac{2}{4}x$$

II. Notation

If $z = f(x, y)$

then $\frac{\partial z}{\partial x} = f_x(x, y) = \frac{d}{dx}\left[f(x, C)\right]$ pretend y is a constant

partial (derivative) with respect to x

f sub x

$\frac{\partial z}{\partial y} = f_y(x, y) = \frac{d}{dy}\left[f(C, y)\right]$

partial (derivative) with respect to y

f sub y

pretend that x is a constant.

III. Examples

(1) $z = f(x, y) = \frac{x^2}{4} - \frac{y^2}{9}$

$$\frac{\partial z}{\partial x} = f_x(x, y) = \frac{1}{4}\cdot 2x + 0 = \frac{1}{2}x$$

$$\frac{\partial z}{\partial y} = f_y(x, y) = 0 - \frac{1}{9}(2y) = -\frac{2}{9}y$$

$$f_x(5,3) = \frac{1}{2}(5) = \frac{5}{2}$$

$$f_y(5,3) = -\frac{2}{9}(3) = -\frac{2}{3}$$

(2) $z = \dfrac{x}{(x+y)^2}$

Pretend y is a constant

$$f_x(x,y) = \frac{(x+y)^2 \cdot 1 - x\, 2(x+y)' \cdot 1}{(x+y)^4}$$

$f_y(x,y)$:

1st step – $f(x,y) = x(x+y)^{-2}$

$$f_y(x,y) = x(-2)(x+y)^{-3} \cdot 1 = \frac{-2x}{(x+y)^3}$$

Might help: $f_y(x,y)$

① Change x to C

$$f(x,y) = C(C+y)^{-2}$$

② Differentiate with respect to y

$$f_y(x,y) = -2C(C+y)^{-3} \cdot 1 = \frac{-2C}{(C+y)^3}$$

③ Change c back to x

$$f_y(x,y) = \frac{-2x}{(x+y)^3}$$

(3) You try! $f(x,y) = \sin(2x)e^{5y}$

Find both $f_x(x,y)$ and $f_y(x,y)$

y is const

$$f_x(x,y) = \boxed{2\cos(2x)e^{5y}}$$

$$f(x,y) = \sin(2x)\underbrace{e^{5y}}_{\text{const.}}$$

x is constant

$$f_y(x,y) = \boxed{\sin(2x)\cdot 5e^{5y}}$$

$$f(x,y) = \underbrace{\sin(2x)}_{\text{constant}}e^{5y}$$

(4) $f(x, y) = 5xy + 2xy^2 + 6x^3y^5$

Find $f_x(x,y)$ and $f_y(x,y)$

$f_x(x,y) = 5y + 2y^2 + 18x^2y^5$

$f(x,y) = 5x(y) + 2x(y^2) + 6x^3(y^5)$

$f_y(x,y) = 5x + 2x\cdot(2y) + 6x^3\cdot 5y^4$

$= 5x + 4xy + 30x^3y^4$

(5) $f(x) = \tan(6x^2y)$

$f_x(x,y) = \sec^2(6x^2y)\cdot 12xy$

$f(x,y) = \tan(6x^2(y))$

$f_y(x,y) = \sec^2(6x^2y)\cdot 6x^2$

$f(x,y) = \tan((6x^2)y)$